Resistance, Reactance and Impedance

One of my Christmas gifts last December was the book "The Forgotten Genius of Oliver Heaviside – A Maverick of Electrical Science". Oliver Heaviside was English (1850 to 1925) and spent most of his life developing his theories alone – a true reclusive genius. I had never heard of him, but it would appear he deserves far greater recognition because he developed many of the theoretical concepts we now take for granted. In one of his early published papers Heaviside reformulated Maxwell's theory of electromagnetism (originally 12 equations that baffled most scientists of his time) into the four Maxwell's Equations now familiar to all electrical engineers. In that same paper published in 1885, Heaviside developed a formula for energy flow through an electromagnetic field that demonstrated electrical energy doesn't flow in a wire but rather in the space alongside the wire. And in 1887 he published the paper "On Resistance and Conductance Operators" which transformed circuit theory with its generalization of Ohm's law. In 1912 he was on the short-list for the Nobel Prize in physics along with Einstein and Planck, but the Prize went to Niels Dalen. Most of Heaviside's work has been collected into five volumes that together total more than 2,500 pages.

The book inspired me to break out my HP50g calculator and review the basics of transmission line theory. The intent in this and several future articles is to review some of these basics that were taught to many of us early on but are now seldom used. Here I begin with resistance, reactance, and impedance. This full report is an expanded version of my *Summer 2020 Broadband Library Testing in Progress* column. This version contains expanded explanations and additional diagrams beyond the space limitations of my normal 2 pages.

Resistance

Resistance as defined by Ohm's Law is "the circuit characteristic that limits current flow." Resistance may arise from a discrete component such as a resistor, or can represent the cumulative opposition to current of wire(s), cable(s), and transmission line(s) including coaxial cable. Resistance, by definition, provides a value in ohms at zero frequency (direct current or DC).

See Figure 1 for Ohm's Law relationships between current, voltage, resistance and power.

When dealing with frequencies greater than zero (alternating current or AC), we find that resistance alone is insufficient to properly quantify the total opposition to current flow in a circuit. This leads to reactance.

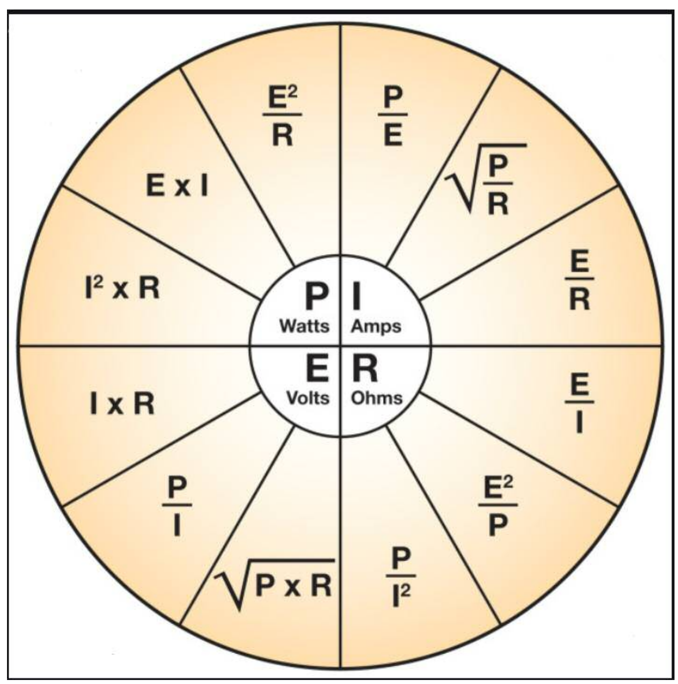

Figure 1 – Ohm's Law Chart

Reactance

Reactance (X) is defined as "that part of the overall opposition to current flow in an AC circuit due to capacitance, inductance, or both, and is expressed in ohms." Reactance has both magnitude and phase values, and further separates into inductive and capacitive components.

Inductive Reactance

Inductive reactance (X_L) is that portion of total circuit reactance contributed by coils, chokes, and transformer windings. Any device in which wire is wound circularly is an inductor. Inductors tend to oppose any change in current over time. This is because current flowing through an inductor generates a magnetic field. In a direct current circuit, the intensity and direction of the magnetic field remain constant. As a result, current flows easily through the inductor since it simply appears as a length of wire. Magnetic fields, by nature, resist change and oppose any change in current. In an AC circuit the magnetic field must change constantly as the magnitude and direction of current flow changes. Thus, an inductor passes lower frequencies more easily than higher frequencies. This opposition to a change in current flow is inductive reactance and is defined by the formula: $\boldsymbol{X_L = 2\pi FL}$ where X_L is inductive reactance in ohms, π is 3.14159(...), F is frequency in hertz, and L is inductance in henrys.

Capacitive Reactance

Capacitive reactance (Xc) is that portion of total circuit reactance caused by capacitance. Capacitance results when two conducting surfaces are parallel to each other and separated by a small distance with a non-conducting substance (dielectric). An example is the discrete circuit component called a capacitor. Capacitors are voltage limiting devices in that they tend to oppose voltage change over time.

When a DC voltage is applied to a capacitor, the capacitor draws a current and charges up to the value of the applied voltage. In an AC circuit, the lower the frequency of the applied voltage, the more time the capacitor has to charge before the voltage reverses polarity and the capacitor begins to discharge. The capacitor therefore spends more time fully charged and passing less current, resulting in less current flow (higher reactance) at low frequencies. As frequency increases, the capacitor changes from charging to discharging faster, allowing more current to flow (lower reactance). Capacitive reactance therefore decreases as frequency increases. The actual value of Xc is inversely proportional to the value of capacitance and the frequency as shown in the formula: $\boldsymbol{X_C = \frac{1}{2\pi FC}}$ where X_C is capacitive reactance in ohms, π is 3.14159(...), F is frequency in hertz, and C is capacitance in farads.

Impedance

Impedance (Z) is the total opposition to current flow in an AC circuit that contains resistance and reactance, and here's where it gets interesting. If a circuit, for example, were purely inductive (no resistance), X_L would represent the total opposition to current flow. In this example Z and X_L are identical and are represented by a value of some magnitude with a -90° phase angle (ɵ or theta). If the circuit were purely capacitive, the same would be true except ɵ shifts to +90°. The phase of the current is always stated relative to the voltage (leading or lagging). AC networks typically contain elements of resistance, inductance and capacitance, and therefore the impedance must be a complex value with magnitude and phase (a vector). The magnitude portion of Z in a circuit can be calculated as shown in the following formulas.

In a series circuit impedance is calculated by $\boldsymbol{Z = \sqrt{R^2 + X_T^2}}$ where $\boldsymbol{X_T = X_L - X_C}$ (X_T is combined circuit reactance).

In a parallel circuit impedance is calculated by $\boldsymbol{Z} = \frac{RX_T}{\sqrt{R^2+X_T^2}}$ where $\boldsymbol{X_T} = \frac{X_L X_C}{X_L - X_C}$.

Now examine the circuit in Figure 2 that contains a combination of resistance and capacitance (no inductance to keep this simple). Note that the resistive and reactive currents do not numerically total as we might expect; 1.0 ampere resistive current plus 1.0 ampere reactive current equals 1.414 amperes total current. This is confusing until we examine Figure 3.

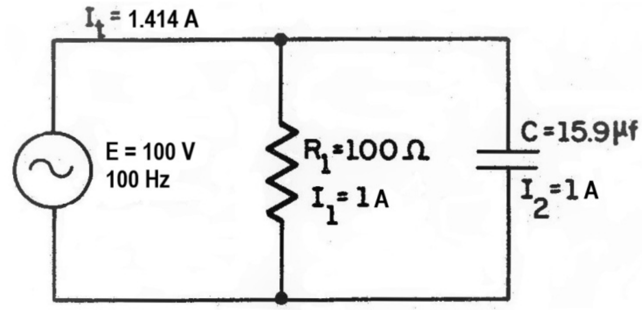

Figure 2 – RC Circuit Diagram

Figure 3 depicts a current triangle as a visual aid to illustrate magnitude and phase. It shows the relationship between the resistive, reactive, and total currents flowing in our sample circuit. If we plot a resistive current of 1 ampere on the x axis (0° theta) and 1 ampere reactive current on the y axis (+90° theta), total current is represented by the length of the hypotenuse at 1.414 amperes with a leading 45°phase angle. Now the current totals make sense!

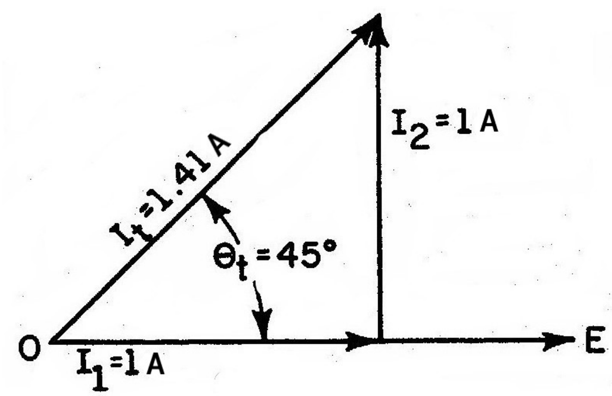

Figure 3 – Current Diagram

Complex Impedance

It's now obvious that impedance and power in this circuit must also be complex (vectors) since current is involved in computing those values. Using the formula for Z (shown earlier) in a parallel circuit yields 70.7 ohms for this RC circuit but doesn't provide the phase angle. An impedance triangle can be constructed for Z, however, the process requires an additional step as we first must calculate an equivalent *series* circuit and then build the triangle.

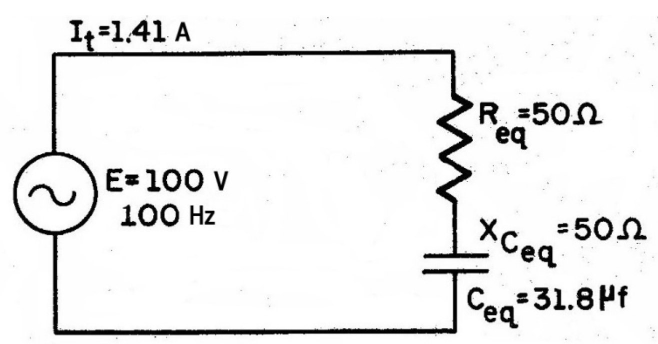

Figure 4 – Equivalent Series Circuit

See Figure 4 for the equivalent series circuit. This is obtained by first calculating the equivalent series resistance value in ohms (2 - 100 ohm resisters in parallel equals 50 ohms in series), then calculating the value of capacitance that yields the required 50 ohms X_C at 100 Hz (because in the actual circuit the value of X_C is 100 ohms in parallel with the resistor). Using the formula for capacitive reactance and solving for C yields 31.831 µf capacitance.

We can now use the equivalent series circuit diagram to obtain the impedance triangle shown in Figure 5. It yields the same magnitude value of Z at 70.7 ohms with a +45° theta, the same phase angle as the current.

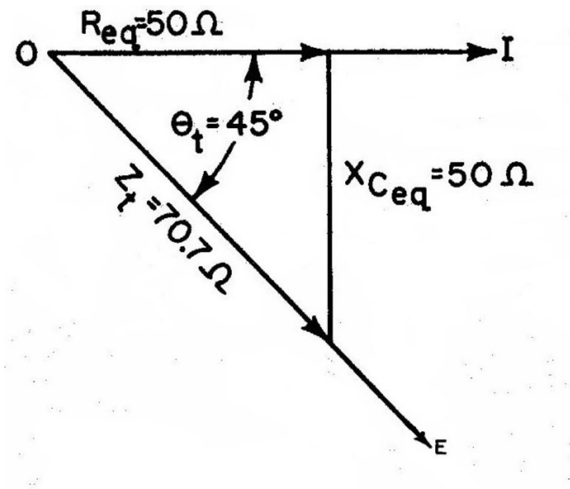

Figure 5 – Impedance Triangle

Finally, in the power triangle shown in Figure 6, the horizontal axis represents true power (resistive, 100 watts), the vertical axis represents the reactive power of 100 VARs (volt-amperes reactive), and the hypotenuse equals the apparent power of 141.4 volt-amperes. Notice that the 45 degree phase angle holds whether we calculate current, impedance or power.

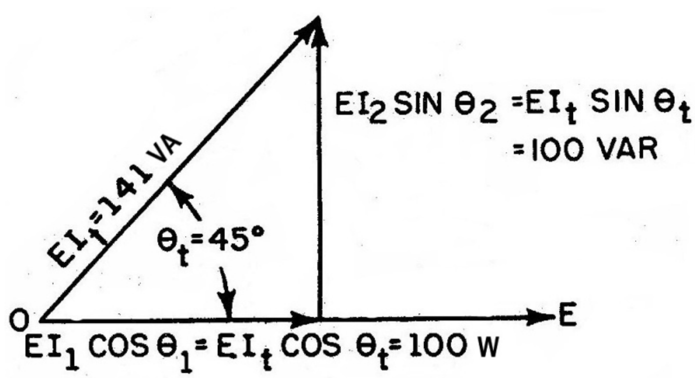

Figure 6 – Power Triangle

Finally, the cosine of the 45° phase angle is 0.707 which is the power factor (*pf*) of this circuit. Power factor is more commonly defined as the ratio of true power (in watts) to apparent power (in volt-amperes). In our sample circuit, 100 watts resistive power divided by 141.4 volt-amperes equals 0.707. Power factor is an important concept in electric utility power distribution networks where their goal is to keep *pf* as close to unity (1) as is possible.

Reactive power is wasted power in an electrical distribution network; it circulates through the network and is dissipated as heat. It is not billed to the customer since kilowatt-hour meters are designed to measure true power only. Most large electrical loads are inductive in nature; transformers, motors, etc. Now examine the formulas for calculating impedance in series and parallel circuits. In both cases, if inductive and capacitive reactance are the same value they cancel each other and what remains, effectively, is a purely resistive circuit. The electric utility can't change the nature of their loads, but what if capacitance could be maintained at just the correct value to cancel network inductance? Electrical distribution networks contain large banks of capacitors at strategic locations for just this purpose. They can be switched in-out of the network to maintain power factor as close to unity as is possible. A utility operator monitors a power factor meter and accomplishes this manually, or in newer networks this may be accomplished automatically. *Pf* is also, in my opinion, an easier way to conceptualize and calculate reactive currents and power.

We'll continue this review in my next column as we move on to parameters specific to coaxial cable.